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Relativistic confinement of neutral fermions with a trigonometric tangent potential

Luis B Castro and Antonio S de Castro

UNESP—Campus de Guaratinguetá, Departamento de Física e Química, Av. Dr. Ariberto P. da Cunha, 333, 12516-410 Guaratinguetá SP, Brazil

E-mail: benito@feg.unesp.br and castro@pesquisador.cnpq.br

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Abstract

The problem of neutral fermions subject to a pseudoscalar potential is investigated. Apart from the solutions for $E = \pm mc^2$, the problem is mapped into the Sturm–Liouville equation. The case of a singular trigonometric tangent potential ($\sim \tan \gamma x$) is exactly solved and the complete set of solutions is discussed in some detail. It is revealed that this intrinsically relativistic and true confining potential is able to localize fermions into a region of space arbitrarily small without the menace of particle–antiparticle production.

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1. Introduction

The four-dimensional Dirac equation with a mixture of spherically symmetric scalar, vector and anomalous magnetic-like (tensor) interactions can be reduced to the two-dimensional Dirac equation with a mixture of scalar, vector and pseudoscalar couplings when the fermion is limited to move in just one direction ($p_y = p_z = 0$) [1]. In this restricted motion the scalar and vector interactions preserve their Lorentz structures while the anomalous magnetic-like interaction becomes a pseudoscalar. This kind of dimensional reduction does not necessarily imply that the reduced Dirac equation describes a fermion in an unrealistic two-dimensional world. As a matter of fact, since there is no spin flip in a one-dimensional motion the two-dimensional version of the Dirac equation can be thought as that one describing a fermion embedded in a four-dimensional space-time with either spin up or spin down [2]. This happens because the four-dimensional Dirac equation with its 4-spinor can be split into two independent Dirac equations with 2-spinors associated with either spin up or spin down. Each one of these 2-spinors has upper and down components associated with particle and antiparticle, respectively. The absence of angular momentum and spin–orbit interaction in the two-dimensional Dirac equation as well as the use of 2×2 matrices, instead of

4×4 matrices, allow us to explore the physical consequences of the negative-energy states in a mathematically simpler and more physically transparent way.

The anomalous magnetic-like (tensor) coupling describes the interaction of neutral fermions with electric fields and the bound states of fermions in one-plus-one dimensions by a pseudoscalar double-step potential [3] and their scattering by a pseudoscalar step potential [4] have already been analysed in the literature providing the opportunity to find some quite interesting results. Indeed, the two-dimensional version of the anomalous magnetic-like interaction linear in the radial coordinate, christened by Moshinsky and Szczepaniak [5] as Dirac oscillator and extensively studied before [6–15], has also received attention. Nogami and Toyama [16], Toyama *et al* [17] and Toyama and Nogami [18] studied the behaviour of wave packets under the influence of that parity-conserving potential whereas Szmytkowski and Gruchowski [19] proved the completeness of the eigenfunctions. More recently Pacheco *et al* [20] studied a few thermodynamics properties of the (1+1)-dimensional Dirac oscillator, and a generalization of the Dirac oscillator for a negative coupling constant was presented in [21]. The two-dimensional generalized Dirac oscillator plus an inversely linear potential has also been addressed [22]. Furthermore, the two-dimensional generalized Dirac oscillator plus scalar and vector harmonic potentials have found a few applications relating nuclear phenomena [23].

The parity-conserving pseudoscalar potential $\sim\tanh\gamma x$ is of interest in quantum field theory where topological classical backgrounds are responsible for inducing a fractional fermion number on the vacuum. Models of these kinds, known as kink models are obtained in quantum field theory as the continuum limit of linear polymer models [24–26]. Recently the complete set of bound states of fermions immersed in the background of the pseudoscalar potential $V = \hbar c\gamma g \tanh\gamma x$, termed kink-like potential, has been addressed [27].

In the present work the pseudoscalar potential $\sim\tan\gamma x$ is investigated. This trigonometric potential has a kink profile in a finite region of space and reveals to be essentially confining. Beyond the confinement property, this potential presents the harmonic oscillator and the infinite square well as limit cases. A peculiar feature of this potential, and for the potential analysed in [27] as well, is the absence of bound states in a nonrelativistic theory because it gives rise to an ubiquitous repulsive potential. Fortunately, apart from solutions corresponding to $|E| = mc^2$, the problem is reducible to the finite set of solutions of the nonrelativistic exactly solvable symmetric Pöschl–Teller potential for both components of the Dirac spinor subject to a constraint on their nodal structure. The whole spectrum of this intrinsically relativistic problem is found analytically, if the fermion is massless or not. A remarkable feature of this problem is the possibility of trapping a fermion with an uncertainty in the position that can shrink without limit as $|\gamma|$ and $|g|$ increase without violating the Heisenberg uncertainty principle. This high degree of localization of fermions in a single-particle interpretation of the theory is made plausible by the introduction of the concept of effective wavelength.

2. The Dirac equation with a pseudoscalar potential in a (1+1) dimension

The (1+1)-dimensional time-independent Dirac equation for a fermion of rest mass m coupled to a pseudoscalar potential reads

$$H\psi = E\psi, \qquad H = c\alpha p + \beta mc^2 + \beta\gamma^5 V, \tag{1}$$

where E is the energy of the fermion, c is the velocity of light and p is the momentum operator. The positive definite function $|\psi|^2 = \psi^\dagger\psi$, satisfying a continuity equation, is interpreted as a position probability density and its norm is a constant of motion. This interpretation is completely satisfactory for single-particle states [28]. We use $\alpha = \sigma_1$ and $\beta = \sigma_3$, where σ_1

and σ_3 are Pauli matrices, and $\beta\gamma^5 = \sigma_2$. The charge conjugation operation requires that if ψ is a solution with eigenenergy E for the potential V then $\sigma_1\psi^*$ is a solution with eigenenergy $-E$ for the potential $-V$. It is interesting to note that the unitary operation just exchanging the upper and lower components of the Dirac spinor induced by $\mathrm{i}\gamma^5$ preserves the eigenenergies for a massless fermion when $V \to -V$. The Dirac equation is covariant under $x \to -x$ if V changes sign. This is because the parity operator $P = \exp(\eta)P_0\sigma_3$, where η is a constant phase and P_0 changes x into $-x$, commutes with σ_3 but anticommutes with σ_1 and σ_2.

Provided that the spinor is written in terms of the upper and the lower components, ψ_+ and ψ_- respectively, the Dirac equation decomposes into

$$(-E \pm mc^2)\psi_\pm = \mathrm{i}\hbar c\psi'_\mp \pm \mathrm{i}V\psi_\mp \tag{2}$$

where the prime denotes differentiation with respect to x. In terms of ψ_+ and ψ_-, defined on the closed interval $[a, b]$, the spinor is normalized as

$$\int_a^b \mathrm{d}x(|\psi_+|^2 + |\psi_-|^2) = 1 \tag{3}$$

so that ψ_+ and ψ_- are square integrable functions.

The boundary conditions on the eigenfunctions come into existence by demanding that the Hamiltonian is Hermitian, viz.

$$\int_a^b \mathrm{d}x\,\psi_n^\dagger(H\psi_{n'}) = \int_a^b \mathrm{d}x(H\psi_n)^\dagger\psi_{n'}, \tag{4}$$

where ψ_n is an eigenspinor corresponding to an eigenvalue E_n. In passing, note that a necessary consequence of equation (4) is that the eigenspinors corresponding to distinct effective eigenvalues are orthogonal. It can be shown that (4) is equivalent to

$$\left[\psi_n^\dagger\sigma_1\psi_{n'}\right]_{x=a}^{x=b} = [(\psi_+^*)_n(\psi_-)_{n'} + (\psi_-^*)_n(\psi_+)_{n'}]_{x=a}^{x=b} = 0. \tag{5}$$

It is clear from the pair of coupled first-order differential equations given by (2) that ψ_+ and ψ_- have definite and opposite parities if the pseudoscalar potential function is odd. In this case, beyond the appropriate boundary conditions on the extremes of the interval, we can impose boundary conditions at the origin in two distinct ways: even functions obey the homogeneous Neumann condition ($\mathrm{d}\psi/\mathrm{d}x|_{x=0} = 0$) whereas odd functions obey the homogeneous Dirichlet condition ($\psi(0) = 0$).

In the nonrelativistic approximation (potential energies small compared to mc^2 and $E \approx mc^2$) equation (2) becomes

$$\psi_- = \left(\frac{p}{2mc} + \mathrm{i}\frac{V}{2mc^2}\right)\psi_+ \tag{6}$$

$$\left(-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{V^2}{2mc^2} + \frac{\hbar V'}{2mc}\right)\psi_+ = (E - mc^2)\psi_+. \tag{7}$$

Equation (6) shows that ψ_- is of order $v/c << 1$ relative to ψ_+ and equation (7) shows that ψ_+ obeys the Schrödinger equation. Note that the pseudoscalar coupling results in the Schrödinger equation with an effective potential in the nonrelativistic limit, and not with the original potential itself. Indeed, this is the side effect which in a (3+1)-dimensional space-time makes the linear potential to manifest itself as a harmonic oscillator plus a strong spin–orbit coupling in the nonrelativistic regime [5]. The form in which the original potential appears in the effective potential, the V^2 term, allows us to infer that even a potential unbounded from below could be a binding potential. This phenomenon is inconceivable if one starts with the original potential in the nonrelativistic equation.

It is also noticeable that the change $V \to V+$ const into the Dirac equation, and into its nonrelativistic limit as well, does not just implies into the change $E \to E+$ const. Strange to say, the energy itself and not just the energy difference has physical significance. It has already been verified that a constant added to the screened Coulomb potential [29] as well as to the inversely linear potential [30] is undoubtedly physically relevant. As a matter of fact, it plays a crucial role to ensure the existence of bound states in those cases.

For $E \neq \pm mc^2$, the coupling between the upper and the lower components of the Dirac spinor can be formally eliminated when equation (2) is written as second-order differential equations:

$$-\frac{\hbar^2}{2}\psi_{\mp}'' + \left(\frac{V^2}{2c^2} \mp \frac{\hbar}{2c}V'\right)\psi_{\mp} = \frac{E^2 - m^2c^4}{2c^2}\psi_{\mp}. \tag{8}$$

Here V is the superpotential corresponding to the Sturm–Liouville supersymmetric partner potentials $V^2/(2c^2) \mp \hbar V'/(2c)$. This supersymmetric structure of the two-dimensional Dirac equation with a pseudoscalar potential has already been appreciated in the literature [18, 31] as has been too for a scalar potential [32]. These last results show that the solution for this class of problem consists in searching for bound-state solutions for two Schrödinger equations. It should not be forgotten, though, that the equations for ψ_+ or ψ_- are not indeed independent because E appears in both equations. Therefore, one has to search for bound-state solutions for both signals in (8) with a common eigenvalue. At this stage one can realize that the Dirac energy levels are symmetrical about $E = 0$. It means that the potential couples to the positive-energy component of the spinor in the same way it couples to the negative-energy component. In other words, this sort of potential couples to the mass of the fermion instead of its charge so that there is no atmosphere for the spontaneous production of particle–antiparticle pairs. Thus there is no room for transitions from positive- to negative-energy solutions. This all means that Klein's paradox never comes to the scenario.

The solutions for $E = \pm mc^2$, excluded from the Sturm–Liouville problem, can be obtained directly from the Dirac equation (2). One can observe that such a sort of isolated solutions can be written as

$$\begin{aligned} &\psi_{\mp} = N_{\mp}\exp[\mp v(x)] \\ &\psi_{\pm}' \mp v'\psi_{\pm} = \pm \mathrm{i}\frac{2mc}{\hbar}N_{\mp}\exp[\mp v(x)], \end{aligned} \tag{9}$$

where N_+ and N_- are normalization constants and $v(x) = \int^x \mathrm{d}y\, V(y)/(\hbar c)$.

The upper and the lower components can be normalized as $\int_a^b \mathrm{d}x|\psi_{\pm}|^2 = |N_{\pm}|^2$ and the absolute values of the relative normalization constants, N_+ and N_-, can be calculated from the Dirac equation (2). Indeed, one has

$$\begin{aligned} (E \pm mc^2)\int_a^b \mathrm{d}x|\psi_{\mp}|^2 &= [(\hbar c)^2\psi_{\pm}^*\psi_{\pm}' \mp \hbar c V|\psi_{\pm}|^2]_{x=a}^{x=b} \\ &\quad + 2c^2\int_a^b \mathrm{d}x\, \psi_{\pm}^*\left(-\frac{\hbar^2}{2}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{V^2}{2c^2} \pm \frac{\hbar}{2c}V'\right)\psi_{\pm} \end{aligned} \tag{10}$$

by imposing boundary conditions which do not break the condition expressed by (5), $\psi_{\pm}(b) = \psi_{\pm}(a) = 0$ for instance, the first term on the right-hand side of (10) vanishes. Hence one can conclude that

$$\int_a^b \mathrm{d}x|\psi_{\pm}|^2 = \frac{E \pm mc^2}{E \mp mc^2}, \qquad \text{for} \quad E \neq \pm mc^2 \tag{11}$$

and

$$-\frac{\hbar^2}{2}\psi_{\mp}'' + \left(\frac{V^2}{2c^2} \mp \frac{\hbar}{2c}V'\right)\psi_{\mp} = 0, \qquad \text{for} \quad E = \pm mc^2. \tag{12}$$

Finally, use of (3) and (11) yields

$$N_{\pm} = \sqrt{\frac{E \pm mc^2}{2E}}. \tag{13}$$

Of course, a possible solution with $E = +mc^2$ ($E = -mc^2$) has a Dirac spinor with a vanishing lower (upper) component. One can observe that such sort of isolated solution for $E = +mc^2$ is

$$\psi \sim \mathrm{e}^{v} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{14}$$

and for $E = -mc^2$ is

$$\psi \sim \mathrm{e}^{-v} \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{15}$$

It is worthwhile to note that whereas one component of the Dirac spinor vanishes the other one obeys a second-order differential equation similar to (8), viz.

$$-\frac{\hbar^2}{2}\psi_{\pm}'' + \left(\frac{V^2}{2c^2} \pm \frac{\hbar}{2c}V'\right)\psi_{\pm} = 0 \qquad \text{and} \qquad \psi_{\mp} = 0. \tag{16}$$

Of course well-behaved eigenstates are possible only if V has an appropriate behaviour at the endpoints of the range $[a, b]$ [27]. It is noticeable that a possible solution with $E = -mc^2$ uncurtains a quintessentially relativistic solution.

3. The trigonometric tangent potential

Now let us concentrate our attention on the potential

$$V = \hbar c\gamma g \tan\gamma x \tag{17}$$

where the kink parameter, γ, and the dimensionless coupling constant, g, are real numbers. Due to the infinities at $x = \pm\pi/(2|\gamma|)$ we restrict ourselves to $|x| \leqslant \pi/(2|\gamma|)$. This potential is unbounded from below so that it is unable to bind a fermion in the nonrelativistic theory. The potential is invariant under the change $\gamma \to -\gamma$ so that the results can depend only on $|\gamma|$ whereas the sign of V depends on the sign of g. Since the solutions for different signs of g can be connected by the charge conjugation transformation, and also by the chiral transformation in the event of massless fermions, we restrict ourselves to the case $g > 0$.

The Sturm–Liouville problem corresponding to equation (8) becomes

$$-\frac{\hbar^2}{2}\psi_{\pm}'' + V_{\mathtt{eff}}^{[\pm]}\psi_{\pm} = E_{\mathtt{eff}}\psi_{\pm}, \tag{18}$$

where we recognize the effective potential as the exactly solvable symmetric Pöschl–Teller potential [33–37]

$$V_{\mathtt{eff}}^{[\pm]}(x) = \frac{\hbar^2\gamma^2}{2}[g(g \pm 1)\tan^2\gamma x \pm g] \tag{19}$$

whose normalizable eigenfunctions corresponding to bound-state solutions, subject to the boundary conditions $\psi_{\pm} = 0$ as $|x| = \pi/(2|\gamma|)$ (where the potential becomes infinitely steep) and identically zero for $|x| > \pi/(2|\gamma|)$, are possible only if the effective potentials for both ψ_{+} and ψ_{-} present potential-well structures. According to (19), this demands that $g > 1$. The corresponding effective eigenenergy is given by (in the notation of [35–37] $g(g \pm 1) = \lambda(\lambda - 1)$)

$$E_{\mathtt{eff}} = \frac{E^2 - m^2c^4}{2c^2} = \frac{\hbar^2\gamma^2}{2}\left(n_{\pm}^2 + 2n_{\pm}\lambda_{\pm} + \lambda_{\pm} \pm g\right), \tag{20}$$

where

$$\lambda_+ = g+1, \qquad \lambda_- = g, \qquad n_\pm = 0,1,2,\ldots. \tag{21}$$

Note that $V_{\mathtt{eff}}^{[\pm]}$ is an even function under $x \to -x$. Furthermore, the capacity of the potential to hold bound-state solutions is infinite. In fact, the effective potential is a well potential limited by infinite barriers at $x = \pi/(2|\gamma|)$. Referring to (19) and (20) one can note that the Dirac eigenenergies are restricted to the range

$$|E| > \sqrt{m^2c^4 + (\hbar c\gamma)^2\, g} \tag{22}$$

and that there is no continuum. Since the positive and negative eigenenergies never intercept once again one can see that Klein's paradox is absent from the scenario. In other words, the pseudoscalar tangent potential is a true confining potential. Furthermore, the fermion tends to be confined into a region of space which tends to zero as $|\gamma| \to \infty$. In order to match the common effective eigenvalue for the effective potentials $V_{\mathtt{eff}}^{[+]}$ and $V_{\mathtt{eff}}^{[-]}$, one can see from (20)–(21) that there appears the constraint

$$n_- = n_+ + 1 \tag{23}$$

requiring that $n_- = 1,2,3,\ldots$ This last fact can be better understood by observing that $V_{\mathtt{eff}}^{[-]}$ is deeper than $V_{\mathtt{eff}}^{[+]}$. Now, (20)–(21) tell us that

$$E = \pm\sqrt{m^2c^4 + (\hbar c\gamma)^2\left[n_+^2 + 2n_+(g+1) + 2g + 1\right]}. \tag{24}$$

The upper and lower components of the Dirac spinor can be written as (see [35–37])

$$\psi_\pm = N_\pm\sqrt{|\gamma|(n_\pm+\lambda_\pm)\frac{\Gamma(2\lambda_\pm+n_\pm)}{\Gamma(n_\pm+1)}}\,(1-z^2)^{1/4}\,P^{1/2-\lambda_\pm}_{n_\pm+\lambda_\pm-1/2}(z), \tag{25}$$

where $z = \sin\gamma x$ and $P_\nu^\mu(z)$ is the associated Legendre function of the first kind. In terms of the Gegenbauer (ultraspherical) polynomial, $C_n^{(a)}(z)$, a polynomial of degree n defined on the interval $[-1,+1]$, the components of the Dirac spinor can be written as

$$\psi_\pm = N_\pm 2^{-\lambda_\pm}\sqrt{2|\gamma|(n_\pm+\lambda_\pm)\frac{\Gamma(n_\pm+1)}{\Gamma(n_\pm+2\lambda_\pm)}}\,\frac{\Gamma(2\lambda_\pm)}{\Gamma(\lambda_\pm+1/2)}(1-z^2)^{\lambda_\pm/2}C_{n_\pm}^{(\lambda_\pm)}(z). \tag{26}$$

Since $C_n^{(a)}(-z) = (-)^n C_n^{(a)}(z)$ and $C_n^{(a)}(z)$ has n distinct zeros (see, e.g. [38]), it becomes clear that ψ_+ and ψ_- have definite and opposite parities, as expected. Furthermore, the number of nodes of ψ_+ and ψ_- just differ by ± 1 according to the rule expressed by (23). Note that these solutions for the second-order differential equations given by (18), for $E \neq -mc^2$, are entirely equivalent to the Dirac equation itself provided $N_\pm$ satisfy equation (22).

It is noteworthy that the width of the position probability density decreases as $|\gamma|$ or g increases. As such it promises that the uncertainty in the position can shrink without limit. It seems that the uncertainty principle dies away provided such a principle implies that it is impossible to localize a particle into a region of space less than half of its Compton wavelength (see, for example, [1]). This apparent contradiction can be remedied by resorting to the concept of effective mass. The previous results suggest that one can define the effective mass as

$$m_{\mathtt{eff}} = \sqrt{m^2 + \left(\frac{\hbar\gamma}{c}\right)^2 g} \tag{27}$$

in such a way that the Dirac eigenenergies are restricted to the range $|E| > m_{\mathtt{eff}}c^2$. Now it is possible to define the effective Compton wavelength as $\lambda_{\mathtt{eff}} = \hbar/(m_{\mathtt{eff}}c)$. Hence, the minimum uncertainty in the position consonant with the uncertainty principle is given by

$\lambda_{\mathtt{eff}}/2$ whereas the maximum uncertainty in the momentum is given by $m_{\mathtt{eff}}c$. It means that the localization of a neutral fermion can shrink to zero without spoiling the single-particle interpretation of the Dirac equation, even if the trapped neutral fermion is massless. It is true that as $|\gamma|$ or g increases the binding potential becomes stronger, though, it contributes to increase the effective mass of the fermion in such a way that there is no energy available to produce fermion–antifermion pairs.

Turning now to the isolated solutions, one can observe from (14) and (15) that a normalizable isolated solution is possible only if the upper component of the spinor vanishes and $E = -mc^2$. The normalized Dirac spinor can be written as

$$\psi = 2^{-g+1/2}\sqrt{|\gamma|g\frac{\Gamma(2g)}{\Gamma(g+1/2)}}(1-z^2)^{g/2}\begin{pmatrix}0\\1\end{pmatrix} \tag{28}$$

independently of the magnitude of g. It turns out that $g > 1$. Indeed, ψ_- satisfies equation (18) with $V_{\mathtt{eff}}^{[-]}$ given by (19) ($E_{\mathtt{eff}} = 0$). Therefore, the coupling constant for an isolated solution has precisely the same restriction as that one for the solutions of the Sturm–Liouville problem. After all, the best localization of fermions as well as the validity of the uncertainty principle is unperturbed if one uses the effective mass given by (27). As a matter of fact, a numerical calculation for the most critical case ($m = 0$) with $g = 1.001$ yields $\Delta x = 0.5680\ \lambda_{\mathtt{eff}}$ and $\Delta p = 0.9995\ m_{\mathtt{eff}}c$, regardless the value of γ ($\hbar = c = 1$).

4. Conclusions

We have succeeded in searching for the complete set of exact bound-state solutions of fermions in the background of a pseudoscalar trigonometric tangent potential. This kind of potential has opposite values at the ends of the interval, viz. $V(+\pi/(2|\gamma|)) = -V(-\pi/(2|\gamma|))$. It is this topological behaviour that gives rise to two different kinds of solutions. The potential admits no scattering states and, except for the solution $E = -mc^2$, it presents a spectral gap greater than $2m_{\mathtt{eff}}c^2$. Since $C_0^{(a)}(z) = 1$ (see, e.g. [38]) and $N_+ = 0$ for $E = -mc^2$, one can see that the position probability amplitude corresponding to the isolated solution given by (28) can be written in the very same mathematical structure of the remaining amplitudes. Thus, one could suspect that the isolated solution is just a particular case and that this segregation is a particularity of the method used in this paper. However, the isolated solution has some distinctive characteristics when compared to the solutions of the Sturm–Liouville problem which lead us to believe that, in fact, they belong to different classes of solutions. The isolated solution breaks the symmetry of the energy levels about $E = 0$ exhibited by the solutions of the Sturm–Liouville problem, and the corresponding eigenspinor has one component differing from zero. It is this asymmetric spectral behaviour that leads to the fractionalization of the fermion number in quantum field theory [26].

For massless fermions, except for $E = 0$, the spectral gap is greater than $2\sqrt{3}\hbar c|\gamma|$ and the Dirac Hamiltonian anticommutes with σ_3 in such a way that the positive- and negative-eigenenergy solutions can be mapped by the operation $\psi_{-E} = \sigma_3\psi_E$. The solution given by (28) appears now in the centre of the spectral gap.

As mentioned in the introduction of this work, the anomalous magnetic-like coupling turns into a pseudoscalar coupling when the fermion experiences a one-dimensional motion. The anomalous magnetic interaction has the form $-\mathrm{i}\mu\beta\vec{\alpha}\cdot\vec{\nabla}\phi(r)$, where μ is the anomalous magnetic moment in units of the Bohr magneton and ϕ is the electric potential, i.e., the time component of a vector potential [28]. In one-plus-one dimensions the anomalous magnetic interaction turns into $\sigma_2\mu\phi'$, then one might suppose that the trigonometric tangent potential is due to an electric potential proportional to $\ln(\cos\gamma x)^g$. Therefore, the problem addressed in this paper could be considered as that one of confining neutral fermions by a bowl-shaped electric potential.

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